KANTOWSKI SACHS DOMAIN WALLS in SAEZ- BALLESTER'S SCALAR- TENSOR THEORIES of GRAVITATION

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Abstract: In this paper general solution are found for domain walls in Saez- Balleters[1] scalar tensor theory of gravitation in the Kantowski Sachs space time. Exact solutions of the field equations are derived when the metric potential are functions of cosmic time only. Some physical and geometric properties of the solutions are also discussed.

Keywords: Saez-Ballester's scalar- tensor theory, Domain wall.

1. INTRODUCTION

Domain walls form when a discrete symmetry is spontaneously broken [2-10]. In simplest models, symmetry breaking is accomplished by a real scalar field ϕ whose vacuum manifold is disconnected. For example, suppose that the scalar potential for ϕ is $U(\phi)=\lambda(\phi^2-\mu^2)^2$. The vacuum manifold for ϕ consists of the two points [$\phi=\mu \quad and \quad \phi=-\mu$]. After symmetry breaking, different regions of the universe can settle into different parts of the vacuum with domain walls forming the boundaries between these regions.. The stress energy for a static, plane symmetry domain wall consists of a positive surface energy density and a surface tension equal in magnitude to the surface energy [5].We note however that this analysis neglects the effect of gravity [11]. Locally, the stress energy for a wall of arbitrary shape is similar to that of a plane symmetric wall having both surface energy density and surface tension.. Closed – surface domain walls collapse due to their surface tension. However, the detail of the collapse for a wall with arbitrary shape and finite thickness are largely unknown.

The investigation of relativistic cosmological models usually has the energy momentum tensor of matter generated by a perfect fluid. To consider more realistic models one must take into account the viscosity mechanisms, which have already attracted the attention of many researchers. Most studies in cosmology involve a perfect fluid. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyse dissipative effects in cosmology.

Based on the cosmological principle, Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the universe since without the cosmological term his field equations admit only non-static cosmological models of non zero energy density. It is well known that a gravitational scalar field, beside the metric of the space –time must exist in the frame work of the present unified theories. Hence there has been much interest in scalar theories of gravitation. Several theories are proposed as an alternative to Einstein's theory to reveal the nature of the universe at the early state of evaluation.

A detailed discussion of Brans- Dicke cosmological models is contained in the work of Singh and Rai[12] while Saez-Ballester[1], Singh and Agrawal [19], Shri Ram and Tiwari[14], Reddy and Venkateswara Rao[15] are some of the authors who have investigated several aspects of Saez-Ballester theory.

In particular, the domain walls have become important in recent years from cosmological stand point when a scenario of galaxy formation has been proposed by [9]. According to them the formation of galaxies are due to domain walls produced during phase transitions after the time of recombination of matter and radiation. So far a considerable amount of work has been done on domain walls. Vilenkin [16], Ipser and Sikivie [17] Windrow [10], Goetz [18], Mukherjee[190], Wang [20], Rahman and Bera [21] Rahman [22], Reddy and Subba Rao [23] are some of the authors who have investigated several aspects of domain walls.

We know that Einstein's field equations are a coupled system of highly non-linear differential equations and we need their physical solutions for its applications in cosmology and astrophysics. In order to find the solutions of the field equations we normally assume a form for the matter content or that space- time admits killing vector symmetries Kramer [24] . The solutions of the field equations can be found by applying a law of variation for

Hubble's parameter which was proposed by Berman [25]. In simplest case the Hubble law yields a constant value for the deceleration parameter

The purpose of the present work is to obtain Kantowski Sachs Domain walls in Saez-Ballester's scalar tensor theories of gravitation. Exact cosmological models are presented with help of special of variation proposed by Barman [25], for Hubble parameter. A static vacuum model and stiff fluid model are presented. The physical and geometrical properties of the models are studied.

2.FIELD EQUATIONS

We consider the spherically symmetric metric in the form

$$ds^2 = -dt^2 + a^2 dr^2 + b^2 d\Omega^2 \tag{1}$$

where a and b are functions of t only.

$$G_i^{\ j} - \omega\phi^n(\phi_{,i}\phi^{,j} - \frac{1}{2}\phi_{,k}\phi^{,k}) = -T_i^{\ j} \tag{2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi_{;i}^{\ i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

here,

$G_i^{\ j} = R_i^{\ j} - \frac{1}{2} g_i^{\ j} R$ is the Einstein tensor,

$T_i^{\ j}$ is the stress tensor of matter,

n is an arbitrary constant,

ω is a dimensionless coupling constant and other symbols have their usual meaning.

Here comma and semicolon denote partial and covariant differentiation respectively.

A thick domain wall can be viewed as solution like solution of the scalar field equations coupled with gravity.

There are two ways of studying thick domain walls.

One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field ψ with self-interactions contained in potential $V(\psi)$ given by

$$\psi_{,i}\psi_{,j} - g_{ij}\left[\frac{1}{2}\psi_{,k}\psi^{,k} - V(\psi)\right] \tag{4}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho(g_{ij} + \omega_i\omega_j) + p\omega_i\omega_j \quad \text{and}$$

$$g^{ij}\omega_i\omega_j = -1 \tag{5}$$

here ,

ρ is the energy density of the wall,

p is the pressure in the direction normal to the plane of the wall and

ω_i is a unit space- like vector in the same direction.

Here we use the second approach to study the thick domain walls in Saez- Ballester theory.

In co-moving co-ordinate system we have from equation (5)

$$T_1^{\ 1} = -p \quad , \quad T_2^{\ 2} = T_3^{\ 3} = T_4^{\ 4} = \rho \quad \text{and}$$

$$T_j^{\ i} = 0 \quad for\ i \neq j \tag{6}$$

Here pressure is taken in the direction of x- axis.

The quantities ρ and p depend on t only.

With the help of equation (4) – (6), the field equations (2) for the metric (1) in the commoving coordinate system take the following explicit forms

$$\frac{2b_{44}}{b} + \frac{1}{b^2} + \frac{b_4^{\ 2}}{b^2} - \omega\phi^n \frac{\phi_4^{\ 2}}{2} = p \tag{7}$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{b_4 a_4}{ba} - \omega\phi^n \frac{\phi_4^{\ 2}}{2} = -\rho \tag{8}$$

$$\frac{2a_4 b_4}{ab} + \frac{1}{b^2} + \frac{b_4^{\ 2}}{b^2} + \omega\phi^n \frac{\phi_4^{\ 2}}{2} = -\rho \tag{9}$$

$$\phi_{44} + \phi_4\left(2\frac{b_4}{b} + \frac{a_4}{a}\right) + \frac{n}{2}\frac{\phi_4^{\ 2}}{\phi} = 0 \tag{10}$$

$$\rho_4 + \frac{a_4}{a}(\rho - p) + 4\rho\frac{b_4}{b} = 0 \tag{11}$$

Here the suffix 4 after field variable represents ordinary differentiation with respect to time.

The physical quantities that are of importance in cosmology are Proper Volume V, expansion scalar θ and shear scalar σ^2 and have the following expressions for the metric

$$V = ab^2 \sin\theta \tag{12}$$

$$\theta = \frac{a_4}{a} + 2\frac{b_4}{b} \tag{13}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{b_4}{b} - \frac{a_4}{a}\right)^2 \tag{14}$$

3. COSMIC DOMAIN WALL in SAEZ-BALLESTER THEORY

Since the field equations are highly non- linear, we also assume an analytic relation between the metric coefficients (scale factor) as

$$a = \mu b \qquad (\mu \text{ is const}) \tag{15}$$

to get determinate solutions.

Now with the help of (15), the field equations of thick domain walls in Saez-Ballester [1] theory can be written as

$$\frac{2b_{44}}{b} + \frac{1}{b^2} + \frac{b_4{}^2}{b^2} - \omega\phi^n \frac{\phi_4{}^2}{2} = p \tag{16}$$

$$2\frac{b_{44}}{b} + \frac{b_4{}^2}{b^2} - \omega\phi^n \frac{\phi_4{}^2}{2} = -\rho \tag{17}$$

$$\frac{1}{b^2} + \frac{3b_4{}^2}{b^2} + \omega\phi^n \frac{\phi_4{}^2}{2} = -\rho \tag{18}$$

$$\phi_{44} + \phi_4\left(3\frac{b_4}{b}\right) + \frac{n}{2}\frac{\phi_4{}^2}{\phi} = 0 \tag{19}$$

$$\rho_4 - p\frac{b_4}{b} + 5\rho\frac{b_4}{b} = 0 \tag{20}$$

Case i): In this case we obtain a cosmological model corresponding to thick domain walls in Saez- Ballester theory with the help of special law of variation for Hubble parameter presented by Barman [25] that Yields constant deceleration parameter models of the universe (The deceleration parameter measures the rate at which expansion of the universe slowing down). We consider only deceleration parameter model defined by

$$q = -\left[\frac{RR_{44}}{(R_4)^2}\right] = const \tag{21}$$

where $R = (ab^2 \sin\theta)^{\frac{1}{3}}$ is the overall scale factor .

Here the constant is taken as negative (i.e. it is an accelerating model of the universe). The solution of (19) is

$$R = (\alpha t + \beta)^{\frac{1}{1+q}} \tag{22}$$

where $\alpha \neq 0$ *and* β are constants of integration. This equation implies that the condition of the expansion $1 + q > 0$ (because the scale factor R can not be negative as well as we know that $q > 0$ then $\frac{dR}{dt}$ is slowing down and if $q < 0$ then

$\frac{dR}{dt}$ is speeding up)

With the help of (21)-(22), the field equations admits an exact solutions

$$a = k_1(\alpha t + \beta)^{\frac{1}{q+1}}; \quad \text{where } k_1 = \mu^{\frac{2}{3}} \sin^{\frac{-1}{3}}\theta \tag{23}$$

$$b = k_2(\alpha t + \beta)^{\frac{1}{q+1}} \quad \text{where } k_2 = \mu^{\frac{1}{3}} \sin^{\frac{-1}{3}}\theta \tag{24}$$

$$\phi^{\frac{n+2}{2}} = \frac{n+2}{2}\left[\frac{c_3}{k_2^3\alpha}\left(\frac{q+1}{q-2}\right)(\alpha t + \beta)^{\frac{q-2}{1+q}} + \phi_0\right] \tag{25}$$

$$p = \frac{\alpha^2(1-2q)}{(1+q)^2(\alpha t + \beta)^2} + \frac{1}{k_2^2(\alpha t + \beta)^{\frac{2}{1+q}}} - \frac{c_3^2\omega}{2(\alpha t + \beta)^{\frac{6}{1+q}}} \tag{26}$$

$$\rho = \frac{c_3^2\omega}{2(\alpha t + \beta)^{\frac{6}{1+q}}} - \frac{\alpha^2(1-2q)}{(1+q)^2(\alpha t + \beta)^2} \tag{27}$$

where k_i's *and* c_3 are constants.

The corresponding model of the solution can be written, through a proper choice of coordinates and constants of integration as

$$ds^2 = -dt^2 + k_1{}^2(\alpha t + \beta)^{\frac{2}{1+q}} dr^2 + k_2^2(\alpha t + \beta)^{\frac{2}{1+q}} d\Omega^2 \tag{28}$$

Using suitable transformation, the metric (28) can be written to the form

$$ds^2 = -\frac{dT^2}{\alpha^2} + T^{\frac{2}{q+2}}\left(dR^2 + d\omega^2\right) \tag{29}$$

Case ii): In order to get the explicit form of physical parameters, we consider here the stiff fluid or self gravitating domain wall with

$$\rho = p \tag{30}$$

With the help of equation (30), equations (16) and (18) yield

$$\frac{b_{44}}{b} + \frac{1}{b^2} + \frac{2b_4{}^2}{b^2} = 0 \tag{31}$$

which leads to

$$2f\frac{df}{db}+4\frac{f^2}{b}=-\frac{2}{b} \quad (32)$$

where $b_4 = f(b)$

Equation (32) lead to

$$f^2=\frac{M}{b^4}-\frac{1}{2} \quad (33)$$

From equation (33), we have

$$dt=\left(\frac{M}{b^4}-\frac{1}{2}\right)^{\frac{-1}{2}}db \quad (34)$$

Then the metric (1) reduce to the form

$$ds^2=-\left(\frac{M}{b^4}-\frac{1}{2}\right)^{-1}db^2+\mu^2b^2dr^2 + b^2d\Omega^2 \quad (35)$$

Using suitable transformation of coordinates, the metric (35) reduce to the form

$$ds^2=-\left(\frac{M}{T^4}-\frac{1}{2}\right)^{-1}dT^2+\mu^2T^2dr^2 + T^2d\Omega^2 \quad (36)$$

where $b=T$

Then the field equations admit an exact solution

$$\phi=\left\{\frac{n+2}{2}\left[\frac{-k_5}{2}\frac{1}{T^2}+k_6\right]\right\}^{\frac{2}{n+2}} \quad (37)$$

$$\rho=p=\frac{1}{2T^2}-\frac{\omega k_1^{\ 2}}{2T^5}-\frac{3M}{T^6} \quad (38)$$

where k_5 *and* k_6 are integrating constants

4.SOME PHYSICAL PROPERTIES

The physical and kinematical quantities for the model (36) have following expressions

Spatial Volume: $V=\mu T^2\sin\theta$ (39)

Expansion Scalar: $\theta=3\left(\frac{M}{T^5}+\frac{-1}{2T}\right)^{\frac{1}{2}}$ (40)

Shear Scalar: $\sigma=0$ (41)

Deceleration

Parameter: $q=-\left[\frac{27}{2}\left(\frac{M}{T^5}-\frac{1}{2T}\right)^{-1}\left(\frac{1}{T^3}-\frac{5M}{T^7}\right)+1\right]$

(42)

Case iii) Here we consider $\rho=p=0$ (vacuum model)

In this case, again assuming the relation between metric coefficient given by the equation (19) and assume the equations (16) –(17) , it is interesting to observe that we get the same model given by (36) with the same physical properties.

5.CONCLUSION

In this paper we have obtained exact solutions in Saez-Ballester theory in the presence of domain walls.

The expression for the scalar scalar field ϕ, pressure p and energy density ρ are given by (25),(26) (27),(37)and (38) respectively.

We observe that at initial moment, when $T=0$ the special volume will be zero while the energy density ρ and pressure p diverge. When $T\to 0$, then expansion scalar θ tends to ∞.

For large values of T , we observe that special volume, expansion scalar θ,pressure and density ρ becomes zero.

Also $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right)=0$ and hence the model approaches isotropy for large value T.

The scalar field ϕ given by equation (25) increases indefinitely as time $T\to\infty$ and is free from initial singularity and the scalar field ϕ given by (38) has the initial singularity.

The model (29) has no initial singularity while pressure p and energy density ρ have initial singularities and the model (30) has no initial singularity while pressure and the energy density ρ have initial singularities. The negative value of the deceleration parameter q given by (36) shows that the model inflate.

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